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Directed Technical Change and Environmental Quality

by

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Biographic Note

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As a member of the University of Porto's team of the student organization *FEP Junior Consulting*, between 2008 and 2009, she developed some consulting projects, integrated in the marketing department, and participated in the organization of two job fair.

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Abstract

We develop a general equilibrium endogenous growth model, which follows the directed technical change literature in a context with: two sectors (clean and dirty), by allowing the dominance of either the price channel or the market-size channel and by considering the possibility of distinct substitutability between sectors in production. The aim is to analyze the impact of both the degree of substitutability between technologies in production and the degree of scale effects on wage inequality and on environmental quality. In particular, we show that when sectors are gross substitutes, an increase in relative abundance of clean-labor supply decreases the relative productivity of clean and dirty technologies. Moreover, an increase in the relative abundance of renewable capital increases the relative productivity of both technologies. The technological knowledge is biased towards the clean sector, i.e., the environmental quality is improved, if the economy is rich in renewable and clean-skilled capital. The relative return on clean-skilled capital increases with the relative abundance of clean-skilled labor under substitutability and without scale effect. The abundance of natural capital increases wage inequality under substitutability.

Keywords: Directed technological change, Clean and dirty sectors, Substitutability, Scale effects, Wage inequality, Environmental quality, Endogenous economic growth.

JEL Classification: O30, O41, J31, Q55, Q58.

Resumo

Desenvolvemos um modelo de crescimento endógeno com equilíbrio geral, que segue a literatura "directed technical change" num contexto com: dois sectores (limpo ou que usa energias renováveis, e sujo ou que usa energias fosseis), permitindo o predomínio do canal preço ou do canal dimensão do mercado, e considerando a possibilidade de substituibilidade distinta entre os setores na produção do bem final compósito. O objetivo é analisar tanto o impacto do grau de substituibilidade entre as tecnologias como o grau de efeitos de escala sobre a desigualdade salarial e a qualidade ambiental. Em particular, mostramos que quando os setores são fortemente substitutos, um aumento na abundância relativa de oferta de trabalho para o sector limpo diminui a produtividade relativa de tecnologias limpas e sujas. Além disso, um aumento na abundância relativa do capital renovável aumenta a produtividade relativa de ambas as tecnologias. O conhecimento tecnológico é enviesado para o sector limpo, isto é, a qualidade ambiental é melhorada, se a economia é rica em capitais renovável e trabalho qualificado para operar no sector limpo. O retorno relativo sobre o capital limpo aumenta com a abundância relativa de mão-de-obra limpa com substituibilidade e sem efeito de escala. A abundância de capital natural aumenta a desigualdade salarial sob substituibilidade.

Palavras-chave: Directed technological change, Setores limpo e sujo, Sustentabilidade, Efeitos de escala, Desigualdade salarial, Qualidade ambiental, Crescimento económico endógeno.

Código JEL: O30, O41, J31, Q55, Q58.

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1. Introduction

Due to growing fossil fuel consumption the climate change is a reality. As a result and to cope with the Kyoto Protocol, the governments have been developing efforts in order to mitigate environmental issues associated to energy. The main idea is to reduce and to control the environmental effects created by firms and consumers. Thus, alternative energy sources to fossil fuels are being studied by climate scientists around the world (Acemoglu *et al.*, 2012). However, the literature on environmental policies tends to give little attention to technological knowledge, by ignoring endogenous technological-knowledge progress. Therefore, to better understand some associated aggregate economic effects, we develop a directed technological change growth model, which extends Acemoglu (1998, 2002) formulation, where two types of technologies can be adopted: a clean technology, represented by renewable or more environmentally friendly sources, and a dirty technology, including fossil fuels.

According to International Energy Agency (IEA, 2015), energy is responsible for 80% of CO2 emission and for 2/3 of total greenhouse gas emissions. Consequently, countries around the world analyze policies capable to limit climate changes and promote a sustainable development. The International Renewable Energy Agency (IRENA, 2015) defends the use of clean technologies. This Agency reveals that, until 2030, clean policies can create around 24 million jobs in the renewable sector. Hence, in a context of moving from fossil fuels to renewable, it is relevant to investigate: (i) the effects of the elasticity of substitution between technologies – clean and dirty; (ii) the scale effects in the technological-knowledge bias, which, in turn, drives the skill premium and the relative return of renewable capital.

The mechanism proposed – technological-knowledge bias – follows a model designed by Acemoglu (2002) and improved by Acemoglu *et al.* (2012). We contribute to this literature by building a general equilibrium endogenous growth model in which the aggregate final good can be produced either in the Clean or in the Dirty sector, being the substitutability between sectors flexible and, in line with the dominant literature on scale effects since Jones (1995a,b), they can be present or not. Thus, firms intend to maximize profits and consumers, the owners of firms, aim at maximize inter-temporal utility. Bearing in mind the connection between intermediate-goods production and the R&D sector, R&D directed to improving Clean intermediate goods can dominate.

We pretend to discuss the impact of both the degree of substitutability between technologies in production and the degree of scale effects on (i) wage inequality and on (ii) environmental quality. It is also possible to analyse the implications for economic growth. That is, our research questions are: what are the implications of endogenous technological-knowledge bias on the relative wages? How does the endogenous bias of the technological knowledge affect the relative return on renewable capital along the BGP? Moreover, we can still, if needed, answer the question: How is economic growth affected?

Contrasting with the inspired models by Acemoglu (2002) and Adu (2012), our approach considers the environment, the endogenous rate and direction of the technological knowledge, the different equations of motions in R&D to address distinct productivities in each sector, and the implication of the market scale, measured by labor levels (Afonso, 2012). Moreover, we present a different formulation comparing Acemoglu *et al.* (2012) model. This model considers that the dirty input is produced using dirty-skilled labor and a natural exhaustible resource, while clean input only uses clean-skilled labor. Our framework, differently, was adapted and extended: there is complementarity between inputs and substitutability between sectors and thus the clean input is produced with clean-skilled labor and a clean resource, i.e., a renewable resource.

According to the literature on the skill-biased technological change, the market-size channel determines the technological-knowledge bias, which motivates wage inequality. Inspired by the literature on scale effects since Jones (1995a, b), the standard R&D technology is rewritten so that scale effects can be removed. In this case, wage inequality results similarly from technological-knowledge bias, which is instead driven by the price channel. According to Acemoglu *et al*. (2016), the discussion of some researches among the transition of dirty to clean technology, in order to propose a reduction of fossil fuels consumption and consequent limitation of CO2 emission, plays a relevant role.

Scott *et al.* (2004), in turn, analyzed that human could influence (increase) the mean summer temperatures concerning a considerable part of Europe. Consequence of a higher sea surface temperatures, tropical cyclones in the Atlantic and western North Pacific cause destruction – see also, for example, Emanuel (2005) and Landsea (2005).

Others, considering empirical evidence, defend that transition as consequence of changes in prices and policies.

Nordhaus (1994) suggests that technological progress and strict control is not sufficient to avoid the massive climate change, consequence of greenhouse gas emissions in the past. The effects of environmental policies in standard studies with exogenous technological knowledge provoke mostly two distinct outputs, which are not optimistic. Nordhaus (2007) suggests limited and slow interventions to decrease long-run growth. In contrast, Stern (2009) proposes wide, stable and abrupt interventions, which comprise significant economic cost.

Newel *et al*. (1999) defend policies that increase the price of oil comparatively to the price of natural gas, because they encourage energy efficiency. Popp (2002) shows that knowledge quality and energy prices affect positively innovation. More recently, Acemoglu *et al.* (2012) explain that a transition to cleaner technology can be a result of carbon taxes and subsidies in research combined. Acemoglu *et al.* (2016) go further and contemplate the optimal set of policies, expecting that carbon taxes give the big contributing due to the incentive of R&D in clean technology and reduction of emissions. As already stated, differently from Acemoglu *et al.* (2012), our model considers a renewable resource in the clean input production and lends much more flexibility to the technological-knowledge bias, allowing scale effects removal. In this case, the effects of the elasticity of substitution between clean and dirty technologies as well as the effects of scale in the technological-knowledge bias, in the premium to human capital (clean-skilled labor) and in the relative return of renewable and fossil capital depend of the relative abundance of renewable capital.

An economic model with two particular sectors was developed, which can be understood as our main contribution: the competitive dirty sector, which uses low-skilled labor and a set of specific intermediate goods, produced under monopolistic competition, and the competitive clean sector, which uses high-skilled labor and a set of specific intermediate goods, also produced under monopolistic competition. The output of the Clean-Sector is produced with clean-skilled labor and a renewable capital. The output of the Dirty-Sector is produced with dirty-skilled labor natural capital and a continuum of labor and natural capital complementary machines. The rate and direction of the technological knowledge are both endogenous, emerging as a result of competitive R&D activities, and then different scenarios are reproduced, according to different elasticity of substitution and the presence (or not) of scale effect.

One of the main results of our model is related to the effect of de elasticity of substitution on technological-knowledge bias. When the factors utilized in both intermediate final good sectors are gross substitutes, an increase in relative abundance of clean-labor supply decreases the relative productivity of clean and dirty technologies. On the other hand, an increase in the relative abundance of renewable capital lends to an increase in the relative productivity of clean and dirty technologies. The direction of technological change is biased towards the clean sector if the economy is rich in renewable and clean-skilled capital. Regarding the premium to clean-skilled capital, the relative return on clean-skilled capital increases with the relative abundance of clean-skilled in the case in which intermediate final good sectors are gross substitutes, and without scale effect. The abundance of natural capital increases wage inequality if the clean sector and the dirty sector are gross substitutes. Finally, the relative return to renewable capital increases in the relative abundance of clean skill if, in particular, the elasticity of substitution between relative abundance of clean-skilled and of renewable capital is greater than one.

Overall, the model built is a two-sector model of directed technological change that can be complemented with more environmental parameters (for example, quality of environmental measures) and with its extension to two regions, allowing for trading between the different Economies.

The rest of the thesis is organized as follows. Section 2 presents the baseline model of directed technological change (extension), the main assumptions and characterizes the equilibrium conditions. Section 3 describes the calibration of the model and shows simulated scenarios and quantitative results. Finally, section 4 presents conclusions and several direction of future research.

2. The Baseline Model

Here, we describe the economic set-up, emphasizing the interactions among economic agents, and the general dynamic equilibrium in which (i) households and firms are rational (solving their problems), (ii) free-entry R&D conditions are met, and (iii) markets clear. We start by considering the optimizing behavior of the infinitely-lived households that inelastically supply labor, clean-skilled or dirty-skilled, maximizes utility of consumption and invests in the firm's equity. Then, we describe the productive side, stressing the baseline maximization problem facing final-good firms, intermediate-good firms and R&D firms.

The inputs of the aggregate (composite, homogenous or consumption) final good (or numeraire) are two intermediate goods, each one composed by a large number of competitive firms: one is produced in the clean-skilled sector and the other is produced in the dirty-skilled sector, and each one uses specific labor and a continuum of specific non-durable intermediate goods. Each intermediate-goods sector consists of a continuum of industries; they are in monopolistic competition if the whole sector is considered: the monopolist in industry j uses a design, sold by the R&D sector (protected, domestically, by a perpetual patent), and aggregate final good to produce, at a price chosen to maximize profits, a non-durable intermediate good. That is, imperfectly competitive firms buy designs (technological knowledge) in the R&D sector to produce intermediate goods, which can complement the inputs used by perfectly competitive final-goods firms in either the clean-skilled sector or the dirty-skilled sector. Therefore, the relative productivity of the technological knowledge depends on the sector in which it is employed. There is free entry in the perfectly competitive R&D sector free entry and every potential entrant dedicates aggregate final good to produce or invent successful horizontal designs. These designs can then be supplied to a new monopolist firm in a new intermediate-goods industry; that is, the R&D sector permits to surge the number of intermediate-goods industries $\boldsymbol{N(t)}$ and thus the technological knowledge.

2.1. Consumers

We assume infinite-lived households deriving utility from consumption C and supply labor inelasticity. Two types of agents are considered among households, according to their competence working or not with "clean" machines: clean-skilled and dirty-skilled workers with, respectively, L_C and L_D aggregate supply. The representative household utility function, considering the populations constant, is:

$$U(C) = \int_0^\infty \frac{C^{1-\sigma}}{1-\sigma} e^{-\theta t} dt \quad (2.1)$$

where θ is the rate of time preference and σ is theinverse of the intertemporal elasticity of substitution. For simplicity of exposition, the time argument in the utility function is suppressed. The aggregate flow budget constraint and the final product market equilibrium condition are equivalent, and the aggregate resource constraint of the economy is defined as follows:

$$\mathrm{Y} \geq C + I + Z \quad (2.1)$$

where Y stands for the aggregate output of the final good sector, I for the total investment and Z for the total R&D expenditure.

Following the no-Ponzi game condition, the lifetime budget constraint of the representative consumer must be satisfied. The asset accumulation equation is:

$$\dot{B} = rB + \gamma\, w_D L_D + (1-\gamma) w_C L_C - C \quad (2.2)$$

where B represents the total household assets and$r$ the share of clean-skilled and dirty-skilled workers in the households, w_C is the wage for clean-skilled workers, L_C, and w_D is the wage for dirty-skilled workers, L_D.

Consumption Euler equation is given by the intertemporal utility maximization problem (See the Appendix for calculations):

$$\frac{\dot{C}}{C} = \frac{1}{\sigma}(r - \rho) \quad (2.3)$$

If the aggregate output and the consumption registry the same rate, the expression (2.4) reveals the aggregate growth rate at stead state.

2.2. Final and intermediate goods production

Two sectors in the country are considered: the clean sector and the dirty sector. The former is clean-skilled and renewable capital intensive, i.e. use non fossil fuels. The latter is dirty-skilled and natural capital intensive, i.e. use fossil fuels. Clean machines can only complement clean-skilled labor and renewable capital while dirty machines can complement dirty-skilled labor and natural capital.

The profit maximization problem will be solved according to final, intermediate and technology firms. In the model, K is interpreted as the renewable capital and R as the natural capital. L_C represents the clean-skilled labor and L_D the dirty-skilled labor.

2.2.1. Maximization problem for final goods products

The final good, which is unique, produced competitively using clean and dirty inputs, respectively, Y_C and Y_D, has the followConstant Elasticity of Substitution (CES) production function:

$$Y = \left[\varsigma_D Y_D^{\frac{\varepsilon-1}{\varepsilon}} + \varsigma_C Y_C^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{2.4}$$

where Y_D isthe output of (input from) the dirty sector (D-Sector),Y_C is the output of (input from) the clean sector (C-Sector) and $\varepsilon \in (0, +\infty)$ is the elasticity of substitution between the two sectors. If $\varepsilon > (<)1$ then the outputs of the D-Sector and the C-Sector are gross substitutes (complements). Normalizing the price of the final good at unit we get:

$$\left[\varsigma_D^{\varepsilon} {P_D}^{1-\varepsilon} + \varsigma_C^{\epsilon} {P_C}^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}} = 1$$

where $[\varsigma_D^{\varepsilon} {P_D}^{1-\varepsilon} + \varsigma_C^{\epsilon} {P_C}^{1-\varepsilon}]^{\frac{1}{1-\varepsilon}}$ is the unit cost of production. The maximization problem is given by:

$$max\, \pi = Y - P_D Y_D - P_C\, Y_C$$

By first order condition we obtain, after some calculations, the relative price of the C-Sector (see appendix for P_C and P_D calculations):

$$\frac{P_C}{P_D} = \frac{\left[\varsigma_D Y_D^{\frac{\varepsilon-1}{\varepsilon}} + \varsigma_C Y_C^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{1}{\varepsilon-1}} \varsigma_C\, Y_C^{-\frac{1}{\varepsilon}}}{\left[\varsigma_D Y_D^{\frac{\varepsilon-1}{\varepsilon}} + \varsigma_C Y_C^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{1}{\varepsilon-1}} \varsigma_D\, Y_D^{-\frac{1}{\varepsilon}}}$$

$$\frac{P_C}{P_D} = \frac{\varsigma_C}{\varsigma_D} \left(\frac{Y_C}{Y_D} \right)^{-\frac{1}{\varepsilon}} \tag{2.5}$$

2.2.2. Maximization problem for intermediate final goods producers

The output of the *D*-Sector is produced with dirty-skilled labor, natural capital and a continuum of labor and natural capital complementary machines x_D in the $[0, A_D]$ interval. The production function for the *D*-Sector is given by:

$$Y_D = \frac{{L_D}^{\alpha} R^{\beta}}{1 - \alpha - \beta} \int_0^{A_D} x_D\,(i)^{1-\alpha-\beta}\,di \tag{2.6}$$

Similarly, the production function for the C-Sector is:

$$Y_C = \frac{{L_C}^{\alpha} K^{\beta}}{1 - \alpha - \beta} \int_0^{A_C} x_C\,(i)^{1-\alpha-\beta} \tag{2.7}$$

where A_D captures the states of the *D*-Sector complementary technologies and A_C captures the state of the *C*-Sector complementary technologies. Maximization problem in the *D*-Sector:

$$max.\pi_D = P_D\,Y_D - w_D L_D - P_R R - \int_0^{A_D} q_D(i) x_D(i)\,di$$

where P_R is the prices of natural capital (R) andq_D is the *D*-Sector complementary machines.

$$max.\pi_D = \frac{{L_D}^{\alpha} R^{\beta}}{1 - \alpha - \beta} \int_0^{A_D} x_D\,(i)^{1-\alpha-\beta}\,di - w_D L_D - P_R R - \int_0^{A_D} q_D\,(i) x_D(i) di$$

From the first order conditions results:

$$\frac{\partial \pi_D}{\partial L_D} = 0$$

$$P_D \frac{\alpha\,{L_D}^{\alpha-1} R^{\beta}}{1-\alpha-\beta} \int_0^{A_D} x_D(i)^{1-\alpha-\beta}\,di - w_D = 0$$

$$\alpha \frac{P_D Y_D}{L_D} - w_D = 0 \tag{2.8}$$

$$\frac{\partial \pi_D}{\partial R} = 0$$

$$P_D \frac{L_D^{\alpha} \beta R^{\beta-1}}{1-\alpha-\beta} \int_0^{A_D} x_D(i)^{1-\alpha-\beta}\,di - P_R = 0$$

$$\beta\,\frac{P_D Y_D}{R} - P_R = 0 \tag{2.9}$$

$$\frac{\partial \pi_D}{\partial x_D(i)} = 0$$

$$P_D \frac{L_D^\alpha R^\beta}{1-\alpha-\beta}(1-\alpha-\beta)x_D(i)^{-\alpha-\beta} - q_D(i) = 0 \text{ (Leibniz integral rue)}$$

$$P_D\, L_D^\alpha R^\beta x_D(i)^{-(\alpha+\beta)} - \; q_D\,(i) = 0 \qquad (2.10)$$

Rearranging equation (2.11) we attain the demand for machine type *i* used in the *D*-Sector:

$$x_D(i)^{-(\alpha+\beta)} = \; \frac{q_D\,(i)}{P_D\, L_D^\alpha R^\beta}$$

$$x_D(i) = \left[\frac{P_D}{q_D(i)} L_D^\alpha R^\beta\right]^{\frac{1}{\alpha+\beta}} \qquad (2.11)$$

We assume that only one type of machine is used in each intermediate good firm (Adu, 2012). Thus, combining (2.12), (2.9) and (2.10) we get the inverse demand functions for labor and natural inputs in the *D*-Sector:

$$w_D = \; \frac{\alpha}{1-\alpha-\beta} P_D{}^{\frac{1}{\alpha+\beta}} q_D(i)^{-\frac{1-\alpha-\beta}{\alpha+\beta}} \left[\frac{L_D}{R}\right]^{-\frac{\beta}{\alpha+\beta}} \qquad (2.12)$$

Similarly:

$$P_R = \frac{\beta}{1-\alpha-\beta} P_D{}^{\frac{1}{\alpha+\beta}} q_D(i)^{-\frac{1-\alpha-\beta}{\alpha+\beta}} \left[\frac{L_D}{R}\right]^{\frac{\alpha}{\alpha+\beta}} \qquad (2.13)$$

The maximization problem in the *C*-Sector is given by:

$$max.\pi_C \; = \; P_C Y_C - w_C\, L_C - P_K K - \int_0^{A_C} q_C(i) x_C(i) di$$

where P_C is the prices of physical capital (K) andq_C is the*C*-Sector complementary machines. From the first order conditions:

$$\frac{\partial \pi_C}{\partial L_C} = \frac{\alpha\, P_C Y_C}{L_C} - w_C \; = 0$$

$$\frac{\partial \pi_C}{\partial K} = \frac{\beta\, P_C Y_C}{K} - P_K = 0$$

$$\frac{\partial \pi_C}{\partial x_C(i)} = P_C L_C^\alpha K^\beta x_C\,(i)^{-(\alpha+\beta)} - q_C(i) = 0$$

Similarly to the previous proof proposition,

$$x_C\,(i) = \left[\frac{P_C}{q_C(i)} L_D^\alpha K^\beta\right]^{\frac{1}{\alpha+\beta}} \qquad (2.14)$$

$$w_C = \frac{\alpha}{1-\alpha-\beta} P_C^{\frac{1}{\alpha+\beta}} q_C(i)^{-\frac{1-\alpha-\beta}{\alpha+\beta}} \left[\frac{L_C}{K}\right]^{\frac{-\beta}{\alpha+\beta}} \tag{2.15}$$

$$P_K = \frac{\beta}{1-\alpha-\beta} P_C^{\frac{1}{\alpha+\beta}} q_C(i)^{-\frac{1-\alpha-\beta}{\alpha+\beta}} \left[\frac{L_C}{K}\right]^{\frac{\alpha}{\alpha+\beta}} \tag{2.16}$$

2.2.3. Maximization problem facing innovators

Regarding the profit maximization problem facing innovators, the machines price, $q(i)$, is chosen by the innovator in order to maximize profits. The marginal cost of building a machine, given by χ, is the same across sectors. Thus, the innovator sells machines at the monopoly price according to the maximization of the profit subject to the demand for machines. The maximization problem for the *D*-Sector is:

$$max.\pi_{D1} = x_D(i)[q_D(i) - \chi]$$

$$max.\pi_{D1} = x_D(i)\left[P_D L_D^{\alpha} R^{\beta} x_D(i)^{-(\alpha+\beta)} - \chi\right]$$

$$max.\pi_{D1} = x_D(i)^{1-\alpha-\beta} P_D L_D^{\alpha} R^{\beta} - x_D(i)\,\chi$$

$$\frac{\partial \pi_{D1}}{\partial x_D(i)} = 0$$

$$(1-\alpha-\beta).x_D(i)^{-(\alpha+\beta)} P_D L_D^{\alpha} R^{\beta} - \chi = 0$$

$$(1-\alpha-\beta).q_D(i) = \chi$$

$$q_D(i) = \frac{\chi}{1-\alpha-\beta}$$

Similarly, for the maximization problem on the *C*-Sector

$$q_C(i) = \frac{\chi}{1-\alpha-\beta} \tag{2.17}$$

Without loss of generality, and according to Acemoglu (2002), it will be assumed that$\chi = 1-\alpha-\beta$.

2.3. Equilibrium Conditions

We consider constant technology combining expressions (2.18), (2.12) and (2.15), we obtain the demand for machines in the two sectors:

$$x_D(i) = \left[P_D L_D^{\alpha} R^{\beta}\right]^{\frac{1}{\alpha+\beta}} \tag{2.18}$$

$$x_C(i) = \left[P_C L_C^{\alpha} K^{\beta}\right]^{\frac{1}{\alpha+\beta}} \tag{2.19}$$

We can rewrite the production function for the intermediate sectors combining equations (2.19), (2.20), (2.7) and (2.8):

$$Y_D = \frac{L_D^{\alpha+1-\alpha-\beta} R^{\beta+1-\alpha-\beta}}{1-\alpha-\beta} P_D{}^{(1-\alpha-\beta)\frac{1}{\alpha+\beta}} L_D^{\alpha\,\frac{1}{\alpha+\beta}1-\alpha-\beta} R^{\beta\,\frac{1}{\alpha+\beta}1-\alpha-\beta}$$
$$Y_D = \frac{L_D^{1-\beta} R^{1-\alpha}}{1-\alpha-\beta}\left[P_D{}^{(1-\alpha-\beta)} L_D^{\alpha} R^{\beta}\right]^{\frac{1}{\alpha+\beta}}$$
$$Y_D = \frac{A_D}{1-\alpha-\beta}\left[P_D{}^{(1-\alpha-\beta)} L_D^{\alpha} R^{\beta}\right]^{\frac{1}{\alpha+\beta}} \tag{2.20}$$

$$Y_C = \frac{A_C}{1-\alpha-\beta}\left[P_C{}^{(1-\alpha-\beta)} L_C^{\alpha} K^{\beta}\right]^{\frac{1}{\alpha+\beta}} \tag{2.21}$$

Rewriting expressions (2.13), (2.14), (2.16) and (2.17), knowing that $\mathrm{q_I} = \frac{\chi}{1-\alpha-\beta} = 1$, we obtain the inverse demand function for *R*:

$$P_R = \frac{\beta}{1-\alpha-\beta} P_D{}^{\frac{1}{\alpha+\beta}} \left(\frac{L_D}{R}\right)^{\frac{\alpha}{\alpha+\beta}} \tag{2.22}$$

the inverse demand function for labor *D*-Sector:

$$W_D = \frac{\alpha}{1-\alpha-\beta} P_D{}^{\frac{1}{\alpha+\beta}} \left(\frac{L_D}{R}\right)^{-\frac{\beta}{\alpha+\beta}} \tag{2.23}$$

the inverse demand function for labor *C*-Sector:

$$W_C = \frac{\alpha}{1-\alpha-\beta} P_C{}^{\frac{1}{\alpha+\beta}} \left(\frac{L_C}{K}\right)^{-\frac{\beta}{\alpha+\beta}} \tag{2.24}$$

and the inverse demand function for *K*:

$$P_K = \frac{\beta}{1-\alpha-\beta} P_C{}^{\frac{1}{\alpha+\beta}} \left(\frac{L_C}{K}\right)^{\frac{\alpha}{\alpha+\beta}} \tag{2.25}$$

The profit of technology monopolist in the *D*-Sector is given by:

$$\pi_D = (\alpha+\beta) P_D^{\frac{1}{\alpha+\beta}} L_D^{\frac{1}{\alpha+\beta}} R^{\frac{1}{\alpha+\beta}} \tag{2.26}$$

and similarly,

$$\pi_C = (\alpha + \beta) P_C^{\frac{1}{\alpha+\beta}} L_C^{\frac{1}{\alpha+\beta}} K^{\frac{1}{\alpha+\beta}} \tag{2.27}$$

By substituting equations (2.21) and (2.22) into (2.6), it's possible to obtain the relative price of the *C*-Factor (see Appendix for deduction):

$$\frac{P_C}{P_D} = \varsigma^{\frac{\varepsilon(\alpha+\beta)}{\sigma}} [\left(\frac{A_C}{A_D}\right)^{\alpha+\beta} \left(\frac{L_C}{L_D}\right)^{\alpha} \left(\frac{K}{R}\right)^{\beta}]^{-\frac{1}{\sigma}} \tag{2.28}$$

where $\sigma = 1 + (\varepsilon - 1)(\alpha + \beta)$ represents the elasticity of substitution between relative abundance of clean-skilled and of renewable capital. It is important to point out that $\varsigma = \frac{\varsigma_C}{\varsigma_D}$ and if $\varepsilon > 1$, $\sigma > 1$ (and vice versa). We can say that the relative price of *C*-Sector output decreases according to the relative abundance of clean-skilled labor $\frac{L_C}{L_D}$ and the relative bias of technology. Nevertheless, it increases in the relative abundance of natural capital $\left(\frac{R}{K}\right)$.Combining equations (2.24), (2.25) and (2.29), we obtain the relative return on clean-skilled capital:

$$\frac{w_C}{w_D} = \left[\varsigma^{-\varepsilon} \left(\frac{A_C}{A_D}\right) \left(\frac{L_C}{L_D}\right)^{\frac{\alpha+\beta\sigma}{\alpha+\beta}} \left(\frac{K}{R}\right)^{\frac{\beta(1-\sigma)}{\alpha+\beta}}\right]^{-\frac{1}{\sigma}} \tag{2.29}$$

Dividing equation (2.26) by (2.23) and combining the resulting equation and equation (2.29), we obtain the relative return of physical capital to natural capital:

$$\frac{P_K}{P_R} = \left[\varsigma^{-\varepsilon} \left(\frac{A_C}{A_D}\right) \left(\frac{L_C}{L_D}\right)^{-\frac{\alpha(\sigma-1)}{\alpha+\beta}} \left(\frac{K}{R}\right)^{\frac{\beta+\alpha\sigma}{\alpha+\beta}}\right]^{-\frac{1}{\sigma}} \tag{2.30}$$

We obtain the relative profitability of innovators in the *C*-Sector dividing equation (2.28) by (2.27) and combining the resulting equation and equation (2.29):

$$\frac{\pi_C}{\pi_D} = \varsigma^{\frac{\varepsilon}{\sigma}} \left(\frac{A_C}{A_D}\right)^{\left(-\frac{1}{\sigma}\right)} \left[\left(\frac{L_C}{L_D}\right)^{\alpha} \left(\frac{K}{R}\right)^{\beta}\right]^{\frac{\sigma-1}{(\alpha+\beta)\sigma}} \tag{2.31}$$

Here it is relevant to denote de $\frac{A_C}{A_D}$role, capable to allow higher profits in the sector with higher price.

2.4. Directed technological change

Now, we analyze how the results of the model are affected by considering that the rate and the direction of the technological knowledge are both endogenous. In the perfectly competitive R&D sector there is free entry and each potential entrant devotes aggregate final good to produce a successful design, which is protected by a system of patents and allows the introduction of a new intermediate good; i.e., a new firm in a new industry *n*. Innovation is revealed by the introduction of new varieties of intermediate inputs with either complements clean-skilled labor (L_C) and renewable capital (K) or dirty-skilled labor (L_D) and natural capital (R). According to Acemoglu (2002, 2008), technologies that complement the dirty and clear sectors change over the time according to the following equations of motions:

$$\dot{A_C} = \lambda_C Z_C L_C^{-\gamma_{L_C}}$$

$$\dot{A_D} = \lambda_D Z_D L_D^{-\gamma_{L_D}} \tag{2.33}$$

Where$\lambda_C$and λ_D are the productivities of the R&D activity in *j*-sector and we assume that $\lambda_C > \lambda_D$. Moreover, $Z_C$and $Z_D$representthe R&D expenditure directed at discovering, respectively, a new *C*-sector and a new *D*-sector augmenting machines. Given that scale effects are often considered implausible (e.g., Jones, 1995a, b), $L_C^{-\gamma_{L_C}}$ and $L_D^{-\gamma_{L_D}}$, implies that an increase in market scale, measured by labor levels weakens the effect of R&D outlays on the innovation rate due to coordination, organizational and transportation costs related to market size (e.g., Afonso, 2012), which, as we can see below, can partially ($0 < \gamma_{L_C}, \gamma_{L_D} < 1$), totally ($\gamma_{L_C} = \gamma_{L_D} = 1$) or over counterbalance ($\gamma_{L_C}, \gamma_{L_D} > 1$) the scale benefits on profits and thus allows us to remove scale effects on the economic growth rate, in contrast with the typical knife-edge hypothesis that either $\gamma_{L_C}, \gamma_{L_D} = 0$ or $\gamma_{L_C}, \gamma_{L_D} = 1$ (e.g., Barro and Sala-i-Martin, 2004, ch. 6).Therefore, total R&D expenditure (Z) satisfies

$$Z = Z_C + Z_D.$$

The Hamilton-Jacobi-Bellman conditions for the value function of a technology monopolist that discovers one of these machines satisfies:

$$RV_j(i) - \dot{V_j}(i) = \pi_j(i), \qquad j = D, C$$

The instantaneous profits for sector $j = D, C$ are specified, respectively, in (2.27) and (2.28) and r is the market interest rate.[1] Since along the balanced growth path (BGP), the interest rate is constant and $\dot{V}_J(i) = 0$,

$$V_D = \frac{\pi_D}{r}$$

$$V_C = \frac{\pi_C}{r} \tag{2.34}$$

where, remember, A_D and A_C do not grow at the same rate at each *t* doubtlessly; this implies that innovators are not indifferent between the sector where they innovate. In particular, remember, $\lambda_C > \lambda_D$, but bearing in mind the free entry condition along the BGP combined with equations (2.32) and (2.29), the relative value for the technology monopolist in the *C*-Sector is (see Appendix):

$$\frac{V_C^*}{V_D^*} = \frac{\lambda_D A_C L_D^{-\gamma_{L_D}}}{\lambda_C A_D L_C^{-\gamma_{L_C}}} = \frac{\pi_C^*}{\pi_D^*} = \varsigma^{\frac{\varepsilon}{\sigma}} \left(\frac{A_C}{A_D}\right)^{\left(-\frac{1}{\sigma}\right)} \left[\left(\frac{L_C}{L_D}\right)^{\alpha} \left(\frac{K}{R}\right)^{\beta}\right]^{\frac{\sigma-1}{(\alpha+\beta)\sigma}} \tag{2.35}$$

Some aspects could be pointed about the relative profitability of technologies related to C-Sector. Indeed, in the case of $\sigma > 1$, there will be an increase in the profitability in the relative abundance of clean-skilled capital and will decrease in the relative abundance of natural capital.

Therefore, see Appendix in order to verify the ratio $\frac{A_C}{A_D}$ deduction:

$$\frac{A_C}{A_D} = \varsigma^{\frac{\varepsilon}{(1+\sigma)}} \left(\frac{\lambda_C}{\lambda_D}\right)^{\frac{\sigma}{(1+\sigma)}} \left(\frac{L_C^{-\gamma_{L_C}}}{L_D^{-\gamma_{L_D}}}\right)^{\frac{\alpha(\sigma-1)}{(1+\sigma)(\alpha+\beta)}} \left(\frac{K}{R}\right)^{\frac{\beta(\sigma-1)}{(1+\sigma)(\alpha+\beta)}} \tag{2.36}$$

Expression (2.36) represents the relative productivity of Clean and Dirty technologies and is the key result of the directed technical change literature. Moreover, in our case is the measure of environmental quality. In the standard directed technical change literature (e.g., Acemoglu 1998, 2002, 2008), the scale has no impact on R&D technology; i.e., scale effects are not removed, $\gamma_{L_C}, \gamma_{L_D} = 0$, and the market-size channel, through which technologies using the more profuse labor type are favored, dominates the chain of effects. In our case, however, the level of scale effects removal lends much more flexibility to the technological-knowledge bias. Thus, $\frac{A_C}{A_D}$ variation if some ratios have a variation of 1 unit, accordingly σ position, as show the Table 1.

[1] In order to simplify the notation, time arguments were dropped.

σ	0	$0 < \sigma < 1$	1	> 1
$\frac{\lambda_C}{\lambda_D}$	0	>0	>0	>0
$\frac{L_C^{-\gamma_{L_C}}}{L_D^{-\gamma_{L_D}}}$	<0	<0	0	>0
$\frac{L_C}{L_D}$	>0	>0	0	<0
$\frac{K}{R}$	<0	<0	0	>0

Table 1. Relative productivity of Clean and Dirty technologies variation if some ratios have a variation of 1 unit, accordingly σ position

Proposition 1. The direction of technological change is endogenized and when $\sigma > 1$, an increase in $\frac{L_C}{L_D}$ will decrease $\frac{A_C}{A_D}$ for a given $\frac{K}{R}$. When $\sigma > 1$, an increase in $\frac{K}{R}$ will increase $\frac{A_C}{A_D}$ for a given $\frac{L_C}{L_D}$, thus improving the environmental quality. Moreover, if the economy is rich, by assumption, in renewable and clean-skilled capital, the direction of technological change becomes biased towards the *C*-sector, cleansing the environment. When $\sigma < 1$, an increase in $\frac{L_C}{L_D}$ will increase $\frac{A_C}{A_D}$ for a given $\frac{K}{R}$ and when $\sigma < 1$, an increase in $\frac{K}{R}$ will decrease $\frac{A_C}{A_D}$ for a given $\frac{L_C}{L_D}$.

Proof. Directly from equation (2.36).

Bearing in mind (2.36), we are able to analyze the implications of endogenous technological-knowledge bias on the relative wage, since, by complementarity between factors in (2.5), changes in wage inequality are tightly connected to the technological-knowledge bias (see equation (2.30)). Hence, we substitute (2.36) into (2.30), we have the skill premium that can be interpreted as the relative return on clean-skilled capital (see Appendix for deduction):

$$W \equiv \frac{w_C}{w_D}$$

$$= \left[\varsigma^{\left(\frac{-\sigma\varepsilon}{1+\sigma}\right)} \left(\frac{\lambda_C}{\lambda_D}\right)^{\left(\frac{\sigma}{1+\sigma}\right)} \left(\frac{L_C^{-\gamma_{L_C}}}{L_D^{-\gamma_{L_D}}}\right)^{\frac{\alpha(\sigma-1)}{(1+\sigma)(\alpha+\beta)}} \left(\frac{L_C}{L_D}\right)^{\frac{\alpha+\beta\sigma}{\alpha+\beta}} \left(\frac{K}{R}\right)^{\frac{\beta(1-\sigma)(1+1+\sigma)}{(1+\sigma)(\alpha+\beta)}} \right]^{-\frac{1}{\sigma}} \tag{2.37}$$

From equation (2.37), W variation if some ratios have a variation of 1 unit, accordingly σ position, as it is shown in the Table 2.

σ	$0 < \sigma < 1$	$\sigma = 1$	$\sigma > 1$
$\frac{\lambda_C}{\lambda_D}$	<0	<0	<0
$\frac{K}{R}$	>0	=0	<0

Table 2. Wages ratio variation if some ratios have a variation of 1 unit, accordingly σ position

Regarding the impact of the ratio $\frac{L_C}{L_D}$, we need to consider the different possibilities of scale effect, as it is represented on the Table 3.

	$0 < \sigma < 1$	$\sigma = 1$	$\sigma > 1$
$\gamma_{L_C} = \gamma_{L_D} = 0$	<0	<0	<0
$0 < \gamma_{L_C} = \gamma_{L_D} < 1$	<0	<0	>0
$\gamma_{L_C} = \gamma_{L_D} = 1$	<0	<0	>0
$\gamma_{L_C} = \gamma_{L_D} > 1$	<0	<0	>0

Table 3. Wages ratio variation if some ratios have a variation of 1 unit, according to scale effect assumption

Proposition 2. Considering $\gamma = \gamma_{L_C} = \gamma_{L_D}$, the expression (2.37) could be rewritten:

$$W = \left[\varsigma^{\left(\frac{-\sigma\varepsilon}{1+\sigma}\right)} \left(\frac{\lambda_C}{\lambda_D}\right)^{\left(\frac{\sigma}{1+\sigma}\right)} \left(\frac{L_C}{L_D}\right)^{\frac{-\gamma\,\alpha(\sigma-1)+(\alpha+\beta\sigma)(1+\sigma)}{(1+\sigma)(\alpha+\beta)}} \left(\frac{K}{R}\right)^{\frac{\beta(1-\sigma)(1+1+\sigma)}{(1+\sigma)(\alpha+\beta)}} \right]^{-\frac{1}{\sigma}}$$

Thus, the relative return on human capital increases in the relative abundance of clean-skilled if $\sigma > 1$and$\gamma_{L_C} = \gamma_{L_D} > 0$. The relative return on clean-skilled capital will increase in the relative abundance of natural capital if $\sigma > 1$. Therefore, if the clean and dirty sectors are gross substitutes in the final good, then the abundance of natural capital increases the wage inequality.

Proof. Directly from equation (2.37).

Then, in order to examine how the endogenous bias of the technological knowledge affects the relative return on K along the BGP, we substitute equation (2.36) into expression (2.31), which can be interpreted as a domestic exchange rate between R, natural capital, and K, renewable capital, or the relative price of K:

$$\frac{P_K}{P_R} = \left[\varsigma^{\frac{-\sigma\varepsilon}{1+\sigma}} \left(\frac{\lambda_C}{\lambda_D}\right)^{\frac{\sigma}{1+\sigma}} \left(\frac{L_C^{-\gamma_{L_C}}}{L_D^{-\gamma_{L_D}}}\right)^{\frac{\alpha(\sigma-1)}{(1+\sigma)(\alpha+\beta)}} \left(\frac{L_C}{L_D}\right)^{\frac{-\alpha(\sigma-1)}{\alpha+\beta}} \left(\frac{K}{R}\right)^{\frac{\beta(\alpha-1)+(\beta+\alpha\sigma)(1+\sigma)}{(1+\sigma)(\alpha+\beta)}} \right]^{-\frac{1}{\sigma}} \qquad (2.38)$$

From equation (2.38), $\frac{P_K}{P_R}$ variation if some ratios have a variation of 1 unit, accordingly σ position, as it is exposed in the Table 4.

σ	$0 < \sigma < 1$	$\sigma = 1$	$\sigma > 1$
$\frac{\lambda_C}{\lambda_D}$	<0	<0	<0
$\frac{K}{R}$	<0	<0	<0

Table 4. Relative return on K variation if some ratios have a variation of 1 unit, accordingly σ position

Once again, regarding the impact of the ratio $\frac{L_C}{L_D}$, it is necessary to consider the different possibilities of scale effect, as it is represented on the Table 5.

	$0 < \sigma < 1$	$\sigma = 1$	$\sigma > 1$
$\gamma_{L_C} = \gamma_{L_D} = 0$	<0	=0	>0
$0 < \gamma_{L_C} = \gamma_{L_D} < 1$	<0	=0	>0
$\gamma_{L_C} = \gamma_{L_D} = 1$	<0	=0	>0
$\gamma_{L_C} = \gamma_{L_D} > 1$	<0	=0	>0

Table 5. Relative return on *K* variation if some ratios have a variation of 1 unit, according to scale effect assumption

Proposition 3. Considering $\gamma = \gamma_{L_C} = \gamma_{L_D}$, the expression (2.38) could be rewritten:

$$\frac{P_K}{P_R} = \left[\varsigma^{\frac{-\sigma\varepsilon}{1+\sigma}}\left(\frac{\lambda_C}{\lambda_D}\right)^{\frac{\sigma}{1+\sigma}}\left(\frac{L_C}{L_D}\right)^{\frac{-\gamma\alpha(\sigma-1)-\alpha(\sigma-1)(1+\sigma)}{(1+\sigma)(\alpha+\beta)}}\left(\frac{K}{R}\right)^{\frac{\beta(\alpha-1)+(\beta+\alpha\sigma)(1+\sigma)}{(1+\sigma)(\alpha+\beta)}}\right]^{-\frac{1}{\sigma}}$$

The relative return to renewable capital rises in the relative abundance of clean-skilled whenever $\sigma > 1$. The relative return to renewable capital will not increase in the relative abundance of natural capital.

Proof. Directly from equation (2.38).

An additional and alternative measure of environmental quality could have been considered, based on the ratio between the production in the clean sector and in the dirty sector. Putting together (2.21) and (2.22), and taking into account expressions (2.29) and (2.36) one gets:

$$\frac{Y_C}{Y_D} = \varsigma^{\frac{\varepsilon\sigma(2-\alpha-\beta)}{(1+\sigma)}}\left(\frac{\lambda_C}{\lambda_D}\right)^{-\frac{1-\alpha-\beta-\sigma}{(1+\sigma)}}\left(\frac{L_C^{(1+\sigma)+(\sigma-1)\gamma_{L_C}}}{L_D^{(1+\sigma)+(\sigma-1)\gamma_{L_D}}}\right)^{-\frac{\alpha(1-\alpha-\beta-\sigma)}{\sigma(1+\sigma)(\alpha+\beta)}}\left(\frac{K}{R}\right)^{-\frac{(1-\alpha-\beta-\sigma)\beta\,(\alpha-1)}{\sigma(\alpha+\beta)}} \quad (2.36)$$

3. Quantitative results

In this section, we examine quantitatively the sensitivity of the principal macroeconomic aggregate ratios of the model to different values for the elasticity of substitution between clean and dirty sectors, given byε, and the scale effects, given by $\gamma = \gamma_{L_C} = \gamma_{L_D}$.

3.1. Calibration

For the quantitative results, we have to set, besides ε and γ, parameters α (the share of labor -clean and dirty - in the intermediate goods production), β (the share of renewable and natural capital in the intermediate goods production), ς_C and ς_D (the distribution or intensity parameter combined to productivity or efficiency of clean and dirty sector, respectively), L_C and L_D (the force employed in the clean and dirty sector, respectively), K (the renewable capital or non-fossil fuels), R (the natural capital or fossil fuels), λ_C and λ_D (the productivities of the E&D activities in the clean and dirty sector, respectively). We set typical values for $\alpha = 0.6$and $\beta = 0.01$.

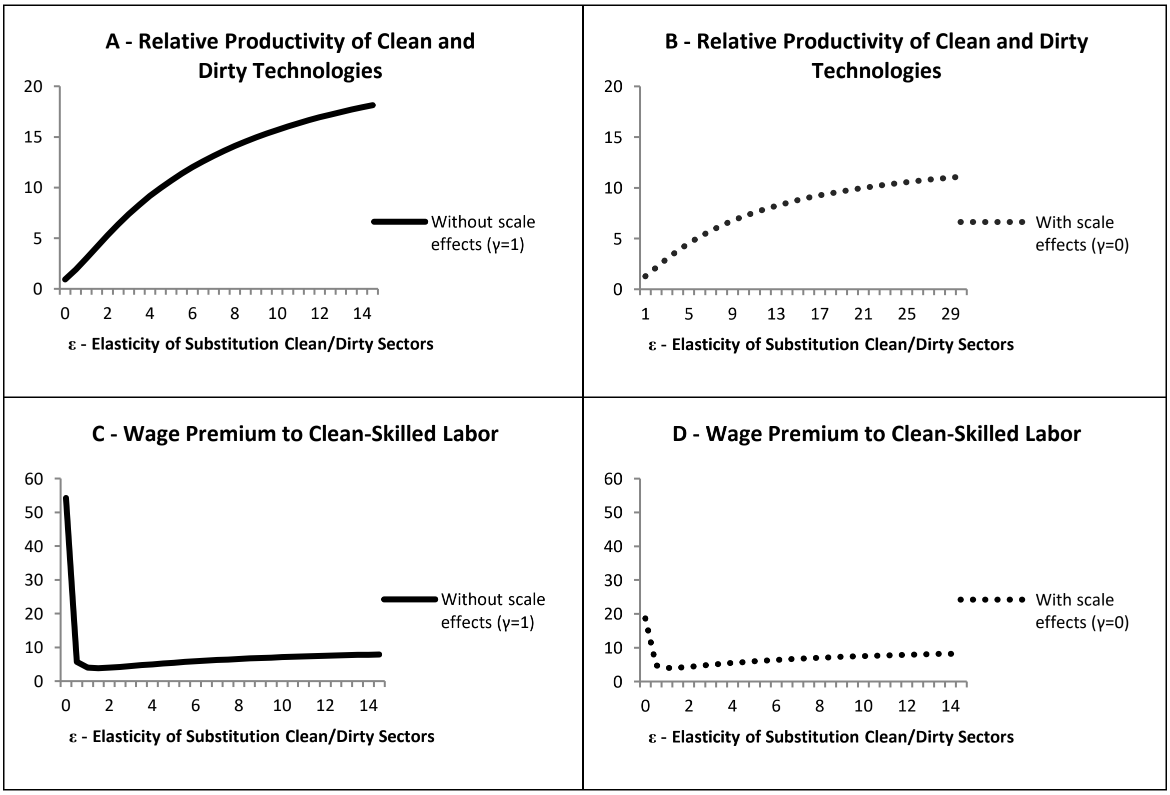
Regarding ς_C and ς_D, we assume that the ratio between ς_C and ς_D, given by ς, is 6, a value higher than 1 that we consider plausible justified by a productivity or efficiency in clean sector higher than in dirty sector. For the ratio $\frac{L_C}{L_D}$ we use a value lower than 1, following the empirical fact of the lower force employed in the clean sector comparing the force employed in the dirty sector. For the ratio $\frac{K}{R}$ we assume as 0.14, a lower value defensible taking in account the lower renewable resources used in the production comparing fossil fuels. Finally, it is acceptable that the ratio between the productivities of the R&D activities in both sectors, $\frac{\lambda_C}{\lambda_D}$, is 1.5 (higher than 1). Table 6 summarizes our calibrated values.

Parameters	Values
α	0.6
β	0.01
ς	6
$\frac{L_C}{L_D}$	0.5
$\frac{K}{R}$	0.14
$\frac{\lambda_C}{\lambda_D}$	1.5

Table 6. Parameters calibrated

3.2. Results

In this section we pretend evaluate how elasticity of substitution between clean and dirty goods, ε, and the presence or not of the scale effect, γ, could influence the main ratios of this directed technical change.



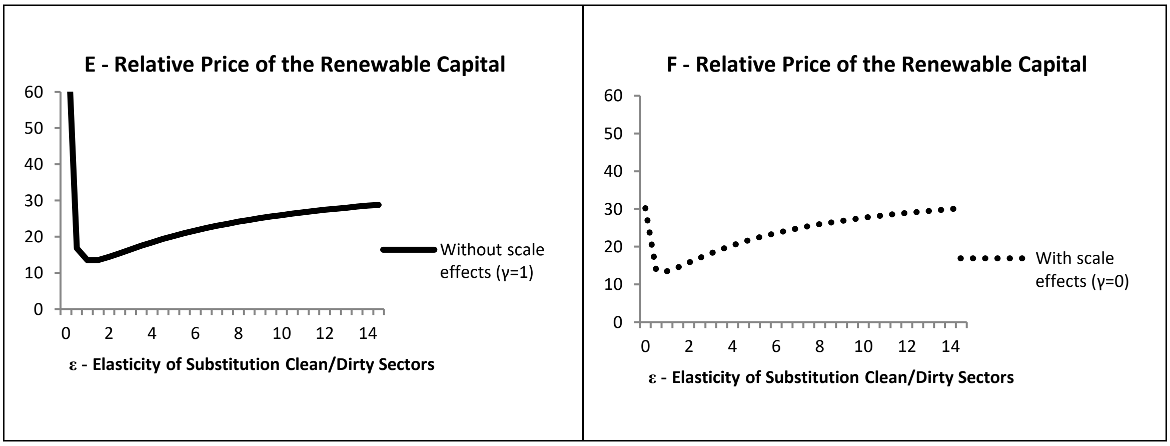


Figure 1. Main ratios for different elasticities of substitution and scale effects

One of the main results of our model is related to the effect of de elasticity of substitution on technological-knowledge bias. Due to the calibration made, considering the absence of scale effects, the variation of the elasticity of substitution between inputs in both sectors will increase the ratio $\frac{A_C}{A_D}$, which is a measure of environmental quality; i.e., as the ε increases and both intermediate final good sectors are being more gross substitutes, it favors the technological-knowledge directed towards the *C*-Sector in the production of intermediate final goods, as we can observe on Figure 1A. The impact of the existence of scale effect could be seen on Figure 1B, and we conclude that the increase of the technological-knowledge bias towards the clean sector is lower under scale effects, but the trend is however maintained.

Regarding wage premium, Figure 1C shows that, without scale effect, while both intermediate final good sectors are complement, the wage inequality decreases significantly, inverting the trend when elasticity of substitution increases and intermediate final good sectors become substitutes. Once again, with scale effects the impact on wage premium is lower than without scale effects, but the trend is maintained (Figure 1D). This result is expected since the technological-knowledge bias affects the wage inequality – see, for example, equation (2.30). Indeed, when the two sectors are strong substitutes, the technological-knowledge is more biased and because of that also the wage inequality increases; i.e., if sectors are strong substitutable, the technological-knowledge bias is more accentuated and thus the productivity of labor in the clean sector increases more. In turn, if sectors are complements, they are both needed in production of the aggregate output and then an increase in the wage in the clean sector requires a greater increase in the wage in the dirty sector.

Now, comparing the two scenarios for the relative price of the renewable capital, with and without scale effects (Figures 1F and 1E), the simulation presents a similar behavior due to the same mechanisms. If the both intermediate final good sectors are complement, the ratio of prices will decrease drastically but if they become substitutes the relative price of the renewable sector will increase.

4. Conclusions

We wanted to examine how both the degree of substitutability between technologies in production and the degree of scale effects could impact on wage inequality and on environmental quality, using for that a directed technical change approach between two sectors (clean and dirty).

A directed technical change endogenous growth model was developed in which the aggregate final good can be produced either in the Clean or in the Dirty sector, with distinct elasticity of substitutability between sectors. Moreover, we also consider different degrees of scale effects. We endogenized the rate and the direction of the technological knowledge, as a result of competitive R&D activities. We conclude that the technological-knowledge bias towards the Clean sector explains the improvement of the environmental quality, which arises, for example, when the economy is rich in renewable and clean-skilled capital.

Moreover, the elasticity of substitution influences the technological-knowledge bias. We can elucidate that if relative abundance of clean-labor supply increase, the relative productivity of clean and dirty technologies will decrease in the case of the factors used in both intermediate final good sectors are gross substitutes. When the relative abundance of renewable capital increases, consequently the relative productivity of clean and dirty technologies increases. Economies rich in renewable and clean-skilled capital assist to the direction of technological change biased towards the clean sector, which is a good sign for the environment quality.

The variation of the elasticity of substitution between inputs in both sectors plays an important role because a positive variation will increase the technological-knowledge bias, what means that if intermediate final good sectors are being more gross substitutes, it favors the technological-knowledge directed towards the Clean sector in the production of intermediate final goods.

Analyzed the premium to clean-skilled capital implications, we conclude that the relative return on clean-skilled capital increases with the relative abundance of clean-skilled in the case in which intermediate final good sectors are gross substitutes, and without scale effect. The abundance of natural capital increases wage inequality if the clean sector and the dirty sector are gross substitutes. Without scale effect, the wage inequality decreases if intermediate final good sectors are complement, inverting the trend when elasticity of substitution increases and intermediate final good sectors become substitutes, an expected result since the technological-knowledge bias affects the wage inequality.

The relative return to renewable capital increases in the relative abundance of clean skill if, in particular, the elasticity of substitution between relative abundance of clean-skilled and of renewable capital is greater than one.

In our approach, the environmental well as the endogenous rate and direction of the technological knowledge are considered. However, it is still limited and should be extended in future research. In particular, it can be complemented with more environmental parameters (for example, quality of environmental measures) and with its extension to two regions, allowing for trading between the different Economies.

Appendix

A. Consumption Euler Equation

From the current value Hamiltonian

$$H = \left(\frac{C^{1-\sigma} - 1}{1-\sigma}\right) + \lambda[rB + \gamma w_D L_D + (1-\gamma) w_C L_C - C]$$

Derivations are made as follows:

$$\frac{\partial H}{\partial C} = C^{-\sigma} - \lambda = 0$$

$$C^{-\sigma} = \lambda$$

$$\frac{\partial H}{\partial B} = \lambda\, r = \rho\,\lambda - \dot{\lambda}$$

$$\dot{\lambda} = (\rho - r)\lambda$$

Derivation of $\frac{\partial H}{\partial C}$ in order to t, the time, yields

$$-\sigma\, C^{-\sigma-1}\dot{C} = \dot{\lambda}$$

Thus,

$$-\sigma\, C^{-\sigma-1}\dot{C} = (\rho - r)\,\lambda$$

$$-\sigma\, C^{-\sigma}C^{-1}\dot{C} = (\rho - r)C^{-\sigma}$$

$$\frac{\dot{C}}{C} = \frac{1}{\sigma}\,(r - \rho) \tag{A.32}$$

B. Final good prices in the two sectors

The maximization problem is given by:

$$max\, \pi = Y - P_D Y_D - P_C\, Y_C$$

$$\pi = \left[\varsigma_D Y_D^{\frac{\varepsilon-1}{\varepsilon}} + \varsigma_C Y_C^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}} - P_D Y_D - P_C\, Y_C$$

Derivation is made:

$$\frac{d\pi}{dY_D} = \frac{\varepsilon}{\varepsilon-1}\left[\varsigma_D Y_D^{\frac{\varepsilon-1}{\varepsilon}} + \varsigma_C Y_C^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}-1} \frac{\varepsilon-1}{\varepsilon}\varsigma_D\, Y_D^{\frac{\varepsilon-1}{\varepsilon}-1} - P_D = 0$$

and the price of the final good in the D-Sector is obtained:

$$P_D = \left[\varsigma_D Y_D^{\frac{\varepsilon-1}{\varepsilon}} + \varsigma_C Y_C^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{1}{\varepsilon-1}} \varsigma_D\, Y_D^{-\frac{1}{\varepsilon}} \tag{A.2}$$

Similarly,

$$P_C = \left[\varsigma_D Y_D^{\frac{\varepsilon-1}{\varepsilon}} + \varsigma_C Y_C^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{1}{\varepsilon-1}} \varsigma_C\, Y_C^{-\frac{1}{\varepsilon}} \tag{A.3}$$

C. Inverse demand for labor derivation

The demand for the demand for machine type i used in the D-Sector is given by:

$$x_D(i) = \left[\frac{P_D}{q_D(i)} L_D^{\alpha} R^{\beta}\right]^{\frac{1}{\alpha+\beta}}$$

$$\frac{\alpha\, P_D Y_D}{L_D} = w_D$$

$$w_D = \alpha\, P_D \frac{1}{L_D} \frac{L_D^{\alpha} R^{\beta}}{1-\alpha-\beta} \int_0^{A_D} x_D(i)^{1-\alpha-\beta}\, di$$

$$w_D = \alpha\, P_D \frac{1}{L_D} L_D^{\alpha} R^{\beta} \frac{1}{1-\alpha-\beta} \int_0^{A_D} P_D^{\frac{1-\alpha-\beta}{\alpha+\beta}} q_D(i)^{-\frac{1-\alpha-\beta}{\alpha+\beta}} L_D^{\frac{\alpha(1-\alpha-\beta)}{\alpha+\beta}} R^{\frac{\beta(1-\alpha-\beta)}{\alpha+\beta}} di$$

$$w_D = \frac{\alpha}{1-\alpha-\beta} P_D^{1+\frac{(1-\alpha-\beta)}{\alpha+\beta}} q_D(i)^{-\frac{1-\alpha-\beta}{\alpha+\beta}} L_D^{-1+\alpha+\frac{\alpha(1-\alpha-\beta)}{\alpha+\beta}} R^{\beta+\frac{\beta(1-\alpha-\beta)}{\alpha+\beta}} di$$

$$w_D = \frac{\alpha}{1-\alpha-\beta} P_D^{\frac{1}{\alpha+\beta}} q_D(i)^{-\frac{1-\alpha-\beta}{\alpha+\beta}} L_D^{-\frac{\beta}{\alpha+\beta}} R^{\frac{\beta}{\alpha+\beta}}$$

$$w_D = \frac{\alpha}{1-\alpha-\beta} P_D^{\frac{1}{\alpha+\beta}} q_D(i)^{-\frac{1-\alpha-\beta}{\alpha+\beta}} \left[\frac{L_D}{R}\right]^{-\frac{\beta}{\alpha+\beta}} \tag{A.4}$$

D. The relative price of the C-Factor deduction

$$\frac{P_C}{P_D} = \frac{\varsigma_C}{\varsigma_D}\left[\frac{Y_C}{Y_D}\right]^{-\frac{1}{\varepsilon}}$$

$$\frac{P_C}{P_D} = \frac{\varsigma_C}{\varsigma_D}\left[\frac{\frac{A_C}{1-\alpha-\beta}\left[P_C{}^{(1-\alpha-\beta)}L_C^{\alpha}K^{\beta}\right]^{\frac{1}{\alpha+\beta}}}{\frac{A_D}{1-\alpha-\beta}\left[P_D{}^{(1-\alpha-\beta)}L_D^{\alpha}R^{\beta}\right]^{\frac{1}{\alpha+\beta}}}\right]^{-\frac{1}{\varepsilon}}$$

$$\frac{P_C}{P_D} = \frac{\varsigma_C}{\varsigma_D}\left(\frac{A_C}{A_D}\right)^{-\frac{1}{\varepsilon}}\left(\frac{P_C}{P_D}\right)^{(1-\alpha-\beta)\frac{1}{\alpha+\beta}\left(-\frac{1}{\varepsilon}\right)}\left(\frac{L_C}{L_D}\right)^{\alpha\frac{1}{\alpha+\beta}\left(-\frac{1}{\varepsilon}\right)}\left(\frac{K}{R}\right)^{\beta\frac{1}{\alpha+\beta}\left(-\frac{1}{\varepsilon}\right)}$$

$$\left(\frac{P_C}{P_D}\right)^{(1-\alpha-\beta)\frac{1}{\alpha+\beta}\left(\frac{1}{\varepsilon}\right)+\frac{\varepsilon}{\varepsilon}} = \varsigma\left(\frac{A_C}{A_D}\right)^{-\frac{1}{\varepsilon}}\left(\frac{L_C}{L_D}\right)^{\alpha\frac{1}{\alpha+\beta}\left(-\frac{1}{\varepsilon}\right)}\left(\frac{K}{R}\right)^{\beta\frac{1}{\alpha+\beta}\left(-\frac{1}{\varepsilon}\right)}$$

$$\left(\frac{P_C}{P_D}\right)^{(1-\alpha-\beta)\frac{1}{\alpha+\beta}+\varepsilon} = \varsigma^{\varepsilon}\left(\frac{A_C}{A_D}\right)^{-1}\left(\frac{L_C}{L_D}\right)^{-\alpha\frac{1}{\alpha+\beta}}\left(\frac{K}{R}\right)^{-\beta\frac{1}{\alpha+\beta}}$$

$$\left(\frac{P_C}{P_D}\right)^{\frac{(1-\alpha-\beta)+\varepsilon(\alpha+\beta)}{\alpha+\beta}} = \varsigma^{\varepsilon}\left(\frac{A_C}{A_D}\right)^{-1}\left(\frac{L_C}{L_D}\right)^{-\frac{\alpha}{\alpha+\beta}}\left(\frac{K}{R}\right)^{-\frac{\beta}{\alpha+\beta}}$$

$$\frac{P_C}{P_D} = \varsigma^{\frac{\varepsilon(\alpha+\beta)}{1+(\varepsilon-1)(\alpha+\beta)}}\left(\frac{A_C}{A_D}\right)^{-\frac{\alpha+\beta}{1+(\varepsilon-1)(\alpha+\beta)}}\left(\frac{L_C}{L_D}\right)^{-\frac{\alpha(\alpha+\beta)}{(\alpha+\beta)+1+(\varepsilon-1)(\alpha+\beta)}}\left(\frac{K}{R}\right)^{-\frac{\beta(\alpha+\beta)}{(\alpha+\beta)+1+(\varepsilon-1)(\alpha+\beta)}}$$

$$\frac{P_C}{P_D} = \varsigma^{\frac{\varepsilon(\alpha+\beta)}{\sigma}}\left(\frac{A_C}{A_D}\right)^{-\frac{\alpha+\beta}{\sigma}}\left(\frac{L_C}{L_D}\right)^{-\frac{\alpha}{\sigma}}\left(\frac{K}{R}\right)^{-\frac{\beta}{\sigma}}$$

$$\frac{P_C}{P_D} = \varsigma^{\frac{\varepsilon(\alpha+\beta)}{\sigma}}\left[\left(\frac{A_C}{A_D}\right)^{\alpha+\beta}\left(\frac{L_C}{L_D}\right)^{\alpha}\left(\frac{K}{R}\right)^{\beta}\right]^{-\frac{1}{\sigma}} \tag{A.5}$$

E. The relative return on clean-skilled capital deduction

$$W = \frac{w_C}{w_D} = \frac{\frac{\alpha}{1-\alpha-\beta}P_C{}^{\frac{1}{\alpha+\beta}}L_C^{\frac{-\beta}{\alpha+\beta}}K^{\frac{\beta}{\alpha+\beta}}}{\frac{\alpha}{1-\alpha-\beta}P_D{}^{\frac{1}{\alpha+\beta}}L_D^{\frac{-\beta}{\alpha+\beta}}R^{\frac{\beta}{\alpha+\beta}}}$$

$$\frac{w_C}{w_D} = \left(\frac{P_C}{P_D}\right)^{\frac{1}{\alpha+\beta}}\left(\frac{L_C}{L_D}\right)^{\frac{-\beta}{\alpha+\beta}}\left(\frac{K}{R}\right)^{\frac{\beta}{\alpha+\beta}}$$

$$\frac{w_C}{w_D} = \varsigma^{\frac{\varepsilon(\alpha+\beta)}{\sigma}\frac{1}{\alpha+\beta}}\left[\left(\frac{A_C}{A_D}\right)^{\alpha+\beta}\left(\frac{L_C}{L_D}\right)^{\alpha}\left(\frac{K}{R}\right)^{\beta}\right]^{-\frac{1}{\sigma}\frac{1}{\alpha+\beta}}\left(\frac{L_C}{L_D}\right)^{\frac{-\beta}{\alpha+\beta}}\left(\frac{K}{R}\right)^{\frac{\beta}{\alpha+\beta}}$$

$$\frac{w_C}{w_D} = \left[\varsigma^{-\varepsilon} \left(\frac{A_C}{A_D}\right) \left(\frac{L_C}{L_D}\right)^{\frac{\alpha+\beta\sigma}{\alpha+\beta}} \left(\frac{K}{R}\right)^{\frac{\beta(1-\sigma)}{\alpha+\beta}} \right]^{-\frac{1}{\sigma}} \tag{A.6}$$

F. The relative return of physical capital to natural capital deduction

$$\frac{P_K}{P_R} = \frac{\frac{\beta}{1-\alpha-\beta} P_C^{\frac{1}{\alpha+\beta}} \left(\frac{L_C}{K}\right)^{\frac{\alpha}{\alpha+\beta}}}{\frac{\beta}{1-\alpha-\beta} P_D^{\frac{1}{\alpha+\beta}} \left(\frac{L_D}{R}\right)^{\frac{\alpha}{\alpha+\beta}}}$$

$$\frac{P_K}{P_R} = \left(\frac{P_C}{P_D}\right)^{\frac{1}{\alpha+\beta}} \left(\frac{L_C}{L_D}\right)^{\frac{\alpha}{\alpha+\beta}} \left(\frac{R}{K}\right)^{\frac{\alpha}{\alpha+\beta}}$$

$$\frac{P_K}{P_R} = \varsigma^{\frac{\varepsilon(\alpha+\beta)}{\sigma}\frac{1}{\alpha+\beta}} \left[\left(\frac{A_C}{A_D}\right)^{\alpha+\beta} \left(\frac{L_C}{L_D}\right)^{\alpha} \left(\frac{K}{R}\right)^{\beta} \right]^{-\frac{1}{\sigma}\frac{1}{\alpha+\beta}} \left(\frac{L_C}{L_D}\right)^{\frac{\alpha}{\alpha+\beta}} \left(\frac{R}{K}\right)^{\frac{\alpha}{\alpha+\beta}}$$

$$\frac{P_K}{P_R} = \varsigma^{-\varepsilon\left(-\frac{1}{\sigma}\right)} \left(\frac{A_C}{A_D}\right)^{-\frac{1}{\sigma}} \left(\frac{L_C}{L_D}\right)^{\frac{\alpha-\alpha\sigma}{\alpha+\beta}\left(-\frac{1}{\sigma}\right)} \left(\frac{K}{R}\right)^{\frac{\beta+\alpha\sigma)}{\alpha+\beta}\left(-\frac{1}{\sigma}\right)}$$

$$\frac{P_K}{P_R} = \left[\varsigma^{-\varepsilon} \left(\frac{A_C}{A_D}\right) \left(\frac{L_C}{L_D}\right)^{-\frac{\alpha(\sigma-1)}{\alpha+\beta}} \left(\frac{K}{R}\right)^{\frac{\beta+\alpha\sigma}{\alpha+\beta}} \right]^{-\frac{1}{\sigma}} \tag{A.7}$$

G. The relative profitability of innovators in the *C*-Sector deduction

$$\frac{\pi_C}{\pi_D} = \frac{(\alpha+\beta) P_C^{\frac{1}{\alpha+\beta}} \left(\frac{L_C}{K}\right)^{\frac{\alpha}{\alpha+\beta}}}{(\alpha+\beta) P_D^{\frac{1}{\alpha+\beta}} \left(\frac{L_D}{R}\right)^{\frac{\alpha}{\alpha+\beta}}}$$

$$\frac{\pi_C}{\pi_D} = \left(\frac{P_C}{P_D}\right)^{\frac{1}{\alpha+\beta}} \left(\frac{L_C}{L_D}\right)^{\frac{\alpha}{\alpha+\beta}} \left(\frac{R}{K}\right)^{\frac{\alpha}{\alpha+\beta}}$$

$$\frac{\pi_C}{\pi_D} = \varsigma^{\frac{\varepsilon(\alpha+\beta)}{\sigma}\frac{1}{\alpha+\beta}} \left[\left(\frac{A_C}{A_D}\right)^{\alpha+\beta} \left(\frac{L_C}{L_D}\right)^{\alpha} \left(\frac{K}{R}\right)^{\beta} \right]^{-\frac{1}{\sigma}\frac{1}{\alpha+\beta}} \left(\frac{L_C}{L_D}\right)^{\frac{\alpha}{\alpha+\beta}} \left(\frac{R}{K}\right)^{\frac{\alpha}{\alpha+\beta}}$$

$$\frac{\pi_C}{\pi_D} = \varsigma^{\frac{\varepsilon}{\sigma}} \left(\frac{A_C}{A_D}\right)^{\frac{\alpha+\beta}{\alpha+\beta}\left(-\frac{1}{\sigma}\right)} \left(\frac{L_C}{L_D}\right)^{\frac{\alpha(\sigma-1)}{\alpha+\beta}\frac{1}{\sigma}} \left(\frac{K}{R}\right)^{\frac{\beta(\sigma-1)}{\alpha+\beta}\frac{1}{\sigma}}$$

$$\frac{\pi_C}{\pi_D} = \varsigma^{\frac{\varepsilon}{\sigma}} \left(\frac{A_C}{A_D}\right)^{\left(-\frac{1}{\sigma}\right)} \left[\left(\frac{L_C}{L_D}\right)^{\alpha} \left(\frac{K}{R}\right)^{\beta}\right]^{\frac{\sigma-1}{(\alpha+\beta)\sigma}} \tag{A.8}$$

H. The relative profitability of innovators in the *C*-Sector deduction and the relative productivity of Clean and Dirty technologies with directed technological change

$$\frac{\dot{A_D}}{A_D} = \frac{\dot{A_C}}{A_C}$$

$$\frac{\lambda_D Z_D L_D^{-\gamma_{L_D}}}{A_D} = \frac{\lambda_C Z_C L_C^{-\gamma_{L_C}}}{A_C}$$

$$\frac{Z_C}{Z_D} = \frac{\lambda_D}{\lambda_C} * \frac{A_C}{A_D} * \frac{L_D^{-\gamma_{L_D}}}{L_C^{-\gamma_{L_C}}}$$

$$\frac{V_C}{V_D} = \frac{Z_C}{Z_D} = \frac{\pi_C}{\pi_D}$$

$$\frac{\lambda_D A_C L_D^{-\gamma_{L_D}}}{\lambda_C A_D L_C^{-\gamma_{L_C}}} = \frac{\pi_C}{\pi_D}$$

$$\frac{V_C^*}{V_D^*} = \frac{\lambda_D A_C L_D^{-\gamma_{L_D}}}{\lambda_C A_D L_C^{-\gamma_{L_C}}} = \frac{\pi_C^*}{\pi_D^*} = \varsigma^{\frac{\varepsilon}{\sigma}} \left(\frac{A_C}{A_D}\right)^{\left(-\frac{1}{\sigma}\right)} \left[\left(\frac{L_C}{L_D}\right)^{\alpha} \left(\frac{K}{R}\right)^{\beta}\right]^{\frac{\sigma-1}{(\alpha+\beta)\sigma}} \tag{A.9}$$

$$\frac{\lambda_D A_C L_D^{-\gamma_{L_D}}}{\lambda_C A_D L_C^{-\gamma_{L_C}}} = \varsigma^{\frac{\varepsilon}{\sigma}} \left(\frac{A_C}{A_D}\right)^{\left(-\frac{1}{\sigma}\right)} \left(\frac{L_C}{L_D}\right)^{\alpha\frac{\sigma-1}{(\alpha+\beta)\sigma}} \left(\frac{K}{R}\right)^{\beta\frac{\sigma-1}{(\alpha+\beta)\sigma}}$$

$$\frac{A_C}{A_D}\left(\frac{A_C}{A_D}\right)^{\left(\frac{1}{\sigma}\right)} = \frac{\lambda_C}{\lambda_D} \varsigma^{\frac{\varepsilon}{\sigma}} \frac{L_C^{-\gamma_{L_C}}}{L_D^{-\gamma_{L_D}}} \left(\frac{L_C}{L_D}\right)^{\alpha\frac{\sigma-1}{(\alpha+\beta)\sigma}} \left(\frac{K}{R}\right)^{\beta\frac{\sigma-1}{(\alpha+\beta)\sigma}}$$

$$\left(\frac{A_C}{A_D}\right)^{\frac{1}{\sigma}+1} = \varsigma^{\frac{\varepsilon}{\sigma}} \frac{\lambda_C}{\lambda_D} \left(\frac{L_C^{-\gamma_{L_C}}}{L_D^{-\gamma_{L_D}}}\right)^{\alpha\frac{\sigma-1}{(\alpha+\beta)\sigma}} \left(\frac{K}{R}\right)^{\beta\frac{\sigma-1}{(\alpha+\beta)\sigma}}$$

$$\left(\frac{A_C}{A_D}\right)^{\frac{(1+\sigma)(\alpha+\beta)}{(\alpha+\beta)\sigma}} = \varsigma^{\frac{\varepsilon(\alpha+\beta)}{(\alpha+\beta)\sigma}} \left(\frac{\lambda_C}{\lambda_D}\right)^{\frac{(\alpha+\beta)\sigma}{(\alpha+\beta)\sigma}} \left(\frac{L_C^{-\gamma_{L_C}}}{L_D^{-\gamma_{L_D}}}\right)^{\alpha\frac{\sigma-1}{(\alpha+\beta)\sigma}} \left(\frac{K}{R}\right)^{\beta\frac{\sigma-1}{(\alpha+\beta)\sigma}}$$

$$\left(\frac{A_C}{A_D}\right)^{(1+\sigma)(\alpha+\beta)} = \varsigma^{\varepsilon(\alpha+\beta)} \left(\frac{\lambda_C}{\lambda_D}\right)^{(\alpha+\beta)\sigma} \left(\frac{L_C^{-\gamma_{L_C}}}{L_D^{-\gamma_{L_D}}}\right)^{\alpha\,(\sigma-1)} \left(\frac{K}{R}\right)^{\beta\,(\sigma-1)}$$

$$\frac{A_C}{A_D} = \varsigma^{\frac{\varepsilon(\alpha+\beta)}{(1+\sigma)(\alpha+\beta)}} \left(\frac{\lambda_C}{\lambda_D}\right)^{\frac{(\alpha+\beta)\sigma}{(1+\sigma)(\alpha+\beta)}} \left(\frac{L_C^{-\gamma_{L_C}}}{L_D^{-\gamma_{L_D}}}\right)^{\frac{\alpha(\sigma-1)}{(1+\sigma)(\alpha+\beta)}} \left(\frac{K}{R}\right)^{\frac{\beta(\sigma-1)}{(1+\sigma)(\alpha+\beta)}}$$

$$\frac{A_C}{A_D} = \varsigma^{\frac{\varepsilon}{(1+\sigma)}} \left(\frac{\lambda_C}{\lambda_D}\right)^{\frac{\sigma}{(1+\sigma)}} \left(\frac{L_C^{-\gamma_{L_C}}}{L_D^{-\gamma_{L_D}}}\right)^{\frac{\alpha(\sigma-1)}{(1+\sigma)(\alpha+\beta)}} \left(\frac{K}{R}\right)^{\frac{\beta(\sigma-1)}{(1+\sigma)(\alpha+\beta)}} \qquad \text{(A.10)}$$

I. The skill premium deduction with directed technical change

$$W \equiv \frac{w_C}{w_D} = \left[\varsigma^{-\varepsilon}\left(\frac{A_C}{A_D}\right)\left(\frac{L_C}{L_D}\right)^{\frac{\alpha+\beta\sigma}{\alpha+\beta}}\left(\frac{K}{R}\right)^{\frac{\beta(1-\sigma)}{\alpha+\beta}}\right]^{-\frac{1}{\sigma}}$$

$$W = \varsigma^{-\varepsilon\left(-\frac{1}{\sigma}\right)}\left[\left(\frac{\lambda_C}{\lambda_D}\right)^{\frac{\sigma}{1+\sigma}} \varsigma^{\frac{\varepsilon}{1+\sigma}} \left(\frac{L_C^{-\gamma_{L_C}}}{L_D^{-\gamma_{L_D}}}\right)^{\frac{\alpha(\sigma-1)}{(1+\sigma)(\alpha+\beta)}} \left(\frac{K}{R}\right)^{\frac{\beta(\sigma-1)}{(1+\sigma)(\alpha+\beta)}}\right]^{-\frac{1}{\sigma}} \left(\frac{L_C}{L_D}\right)^{\frac{\alpha+\beta\sigma}{\alpha+\beta}\left(-\frac{1}{\sigma}\right)} \left(\frac{K}{R}\right)^{\frac{\beta(1-\sigma)}{(\alpha+\beta)}\left(-\frac{1}{\sigma}\right)}$$

$$W = \varsigma^{\left(-\frac{1}{\sigma}\right)\left(-\varepsilon+\frac{\varepsilon}{1+\sigma}\right)} \left(\frac{\lambda_C}{\lambda_D}\right)^{-\frac{1}{\sigma}\left(\frac{\sigma}{1+\sigma}\right)} \left(\frac{L_C^{-\gamma_{L_C}}}{L_D^{-\gamma_{L_D}}}\right)^{\frac{\alpha(\sigma-1)}{(1+\sigma)(\alpha+\beta)}\left(-\frac{1}{\sigma}\right)} \left(\frac{L_C}{L_D}\right)^{\frac{\alpha+\beta\sigma}{\alpha+\beta}\left(-\frac{1}{\sigma}\right)} \left(\frac{K}{R}\right)^{-\frac{1}{\sigma}\left(\frac{\beta(\sigma-1)}{(1+\sigma)(\alpha+\beta)}+\frac{\beta(1-\sigma)}{(\alpha+\beta)}\right)}$$

$$W = \varsigma^{\left(-\frac{1}{\sigma}\right)\left(\frac{-\sigma}{1+\sigma}\right)} \left(\frac{\lambda_C}{\lambda_D}\right)^{-\frac{1}{\sigma}\left(\frac{\sigma}{1+\sigma}\right)} \left(\frac{L_C^{-\gamma_{L_C}}}{L_D^{-\gamma_{L_D}}}\right)^{\frac{\alpha(\sigma-1)}{(1+\sigma)(\alpha+\beta)}\left(-\frac{1}{\sigma}\right)} \left(\frac{L_C}{L_D}\right)^{\frac{\alpha+\beta\sigma}{\alpha+\beta}\left(-\frac{1}{\sigma}\right)} \left(\frac{K}{R}\right)^{-\frac{1}{\sigma}\left(\frac{\beta(1-\sigma)(1+1+\sigma)}{(1+\sigma)(\alpha+\beta)}\right)}$$

$$W = \left[\varsigma^{\left(\frac{-\sigma\varepsilon}{1+\sigma}\right)} \left(\frac{\lambda_C}{\lambda_D}\right)^{\left(\frac{\sigma}{1+\sigma}\right)} \left(\frac{L_C^{-\gamma_{L_C}}}{L_D^{-\gamma_{L_D}}}\right)^{\frac{\alpha(\sigma-1)}{(1+\sigma)(\alpha+\beta)}} \left(\frac{L_C}{L_D}\right)^{\frac{\alpha+\beta\sigma}{\alpha+\beta}} \left(\frac{K}{R}\right)^{\frac{\beta(1-\sigma)(1+1+\sigma)}{(1+\sigma)(\alpha+\beta)}}\right]^{-\frac{1}{\sigma}} \qquad \text{(A.11)}$$

J. The relative return on renewable capital with directed technical change

$$\frac{P_K}{P_R} = \left[\varsigma^{-\varepsilon}\left(\frac{A_C}{A_D}\right)\left(\frac{L_C}{L_D}\right)^{\frac{-\alpha(\sigma-1)}{\alpha+\beta}}\left(\frac{K}{R}\right)^{\frac{\beta(\alpha+1)}{\alpha+\beta}}\right]^{-\frac{1}{\sigma}}$$

$$\frac{P_K}{P_R} = \left[\varsigma^{-\varepsilon} \left(\frac{\lambda_C}{\lambda_D}\right)^{(\frac{\sigma}{1+\sigma})} \varsigma^{\frac{\varepsilon}{1+\sigma}} \left(\frac{L_C^{-\gamma_{L_C}}}{L_D^{-\gamma_{L_D}}}\right)^{\frac{\alpha(\sigma-1)}{(1+\sigma)(\alpha+\beta)}} \left(\frac{K}{R}\right)^{\frac{\beta(\sigma-1)}{(1+\sigma)(\alpha+\beta)}} \left(\frac{L_C}{L_D}\right)^{\frac{-\alpha(\sigma-1)}{\alpha+\beta}} \left(\frac{K}{R}\right)^{\frac{\beta+\alpha\sigma}{(\alpha+\beta)}} \right]^{-\frac{1}{\sigma}}$$

$$\frac{P_K}{P_R} = \left[\varsigma^{\frac{-\sigma\varepsilon}{1+\sigma}} \left(\frac{\lambda_C}{\lambda_D}\right)^{\frac{\sigma}{1+\sigma}} \left(\frac{L_C^{-\gamma_{L_C}}}{L_D^{-\gamma_{L_D}}}\right)^{\frac{\alpha(\sigma-1)}{(1+\sigma)(\alpha+\beta)}} \left(\frac{L_C}{L_D}\right)^{\frac{-\alpha(\sigma-1)}{\alpha+\beta}} \left(\frac{K}{R}\right)^{\frac{\beta(\alpha-1)+(\beta+\alpha\sigma)(1+\sigma)}{(1+\sigma)(\alpha+\beta)}} \right]^{-\frac{1}{\sigma}} \qquad \text{(A.12)}$$

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